Parallel Computing of 3-D Eddy Current Analysis with A-ϕ Method for Rotating Machines

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Abstract — We developed a parallel computing method for rapid eddy current analysis using the *A*-ϕ method. In this paper, the outline of the developed method is described. Moreover, the performance of the proposed method running on a PC cluster is quantitatively clarified through the analysis of an interior permanent magnet motor.

I. INTRODUCTION

The concern with parallel computing has been growing for the last several years. However, there are very few studies, which employ the parallel computing methods on the magnetic field analysis for actual rotating machines.

We have developed a parallel computing method for rotating machines based on the domain decomposition method (DDM) using the three-dimensional finite element method (3-D FEM) formulated by the ***A*** method [1], [2].

For rapid eddy current analysis, we propose a parallel computing method using the *A*-ϕ method. In this paper, performance of the method running on a PC cluster is quantitatively clarified through the analysis of an interior permanent magnet (IPM) motor.

II. ANALYSIS METHOD

A. Fundamental Equations

The fundamental equations of the magnetic field are given by the magnetic vector potential ***A*** and the electric scalar potential ϕ as follows:

$$\mathrm{curl}\left(\nu\,\mathrm{curl}\,\boldsymbol{A}\right) = \boldsymbol{J}_0 + \boldsymbol{J}_e + \nu_0\,\mathrm{curl}\,\boldsymbol{M} \tag{1}$$

$$\boldsymbol{J}_e = -\sigma\left(\frac{\partial \boldsymbol{A}}{\partial t} + \mathrm{grad}\,\phi\right) \tag{2}$$

$$\mathrm{div}\,\boldsymbol{J}_e = 0 \tag{3}$$

where ν is the reluctivity, $\boldsymbol{J}_0$ is the exiting current density, $\boldsymbol{J}_e$ is the eddy current density, ν_0 is the reluctivity of the vacuum, ***M*** is the magnetization of permanent magnet, σ is the electric conductivity.

B. Parallel Computing Based on Domain Decomposition Method for A-ϕ Method

We adopted the overlapping DDM for the parallel computing, because of the robustness and the simplicity. Adopting the DDM, the analyzed domain is divided into multiple subdomains. These subdomains are calculated in parallel with appropriate data communications between them.

Fig. 1 shows an example of the overlapping DDM for the FEM with the *A*-ϕ method, in which whole domain as shown Fig. 1 (a) is divided into two subdomains. The overlapping elements are added to the subdomains in order to communicate potentials ***A*** on edges and ϕ on nodes between neighboring subdomains as shown in Fig. 1 (b). The number of edges in each subdomain is almost the same to split the calculation load into each PC evenly.

The overlapping elements have two types of edges, one is “boundary edge” and the other is “external edge”, and has two types of nodes, one is “boundary node” and the other is “external node” as shown Fig. 1 (b). The boundary edges and nodes in one subdomain are the external edges and nodes in the other subdomain, respectively. In each iteration of Conjugate Gradient method, the potentials on the boundary edges and nodes in a subdomain are sent to neighboring subdomain as the potentials on external edges and nodes.

The matrix equation in the *i*-th subdomain is given as follows:

$$\begin{bmatrix} \left[\dfrac{\partial G^{(i)}}{\partial A^{(i)}}\right] & \left[\dfrac{\partial G^{(i)}}{\partial \phi^{(i)}}\right] \\ \left[\dfrac{\partial G_d^{(i)}}{\partial A^{(i)}}\right] & \left[\dfrac{\partial G_d^{(i)}}{\partial \phi^{(i)}}\right] \end{bmatrix} \begin{Bmatrix} \left\{\delta A^{(i)}\right\} \\ \left\{\delta \phi^{(i)}\right\} \end{Bmatrix} = -\begin{Bmatrix} \left\{G^{(i)}\right\} \\ \left\{G_d^{(i)}\right\} \end{Bmatrix} \tag{4}$$

where $\{G^{(i)}\}$ and $\{G_d^{(i)}\}$ are residuals obtained from equations (1)–(3) with the Galerkin method.

Fig. 2 shows the storage format of the matrix equations divided into two subdomains corresponding to (4). The PC solving the *i*-th subdomain stores only the data whose superscript number is *i*. In order to calculate each subdomain, the potentials on external edges and nodes are obtained from neighboring subdomains by the data communications.

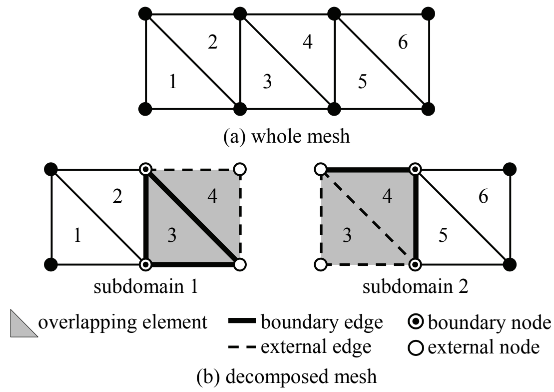
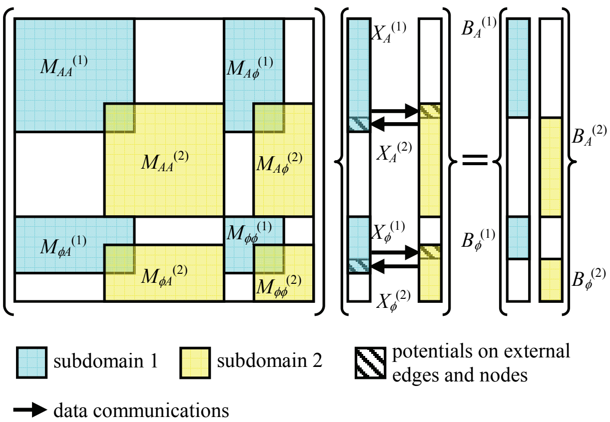


Fig. 1. Domain decomposition.



$$M_{AA}^{(i)} = [\partial G^{(i)}/\partial A^{(i)}] \qquad X_A^{(i)} = \{\delta A^{(i)}\}$$
$$M_{A\phi}^{(i)} = [\partial G^{(i)}/\partial \phi^{(i)}] \qquad X_\phi^{(i)} = \{\delta \phi^{(i)}\}$$
$$M_{\phi A}^{(i)} = [\partial G_d^{(i)}/\partial A^{(i)}] \qquad B_A^{(i)} = \{-G^{(i)}\}$$
$$M_{\phi\phi}^{(i)} = [\partial G_d^{(i)}/\partial \phi^{(i)}] \qquad B_\phi^{(i)} = \{-G_d^{(i)}\}$$

Fig. 2. Storage format of matrix equation with A-ϕ method.

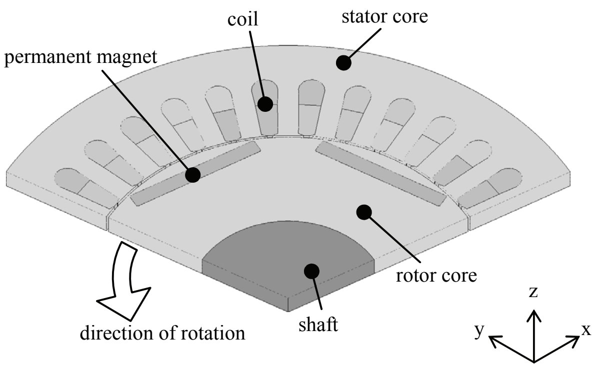


Fig. 3. Analyzed model.

III. NUMERICAL RESULTS AND DISCUSSION

A. Analyzed Model and Analysis Conditions

Fig. 3 shows the analyzed model of an IPM motor. The analyzed region is 1/66 of the whole region because of the symmetry. The length of axial direction in the analyzed region is the same as half of height of a piece of the permanent magnet. The analysis is carried out taking into account the eddy current in the permanent magnet with the A-ϕ method on a PC cluster composed of 8 PCs. Each PC has an Intel Core i7 processor (2.93GHz). The number of elements is 1,100,862.

B. Results and Discussion

Fig. 4 shows the distribution of flux density vectors. The distribution agrees completely with that calculated by one PC.

Fig. 5. shows the distribution of the eddy current loss in the permanent magnet. The distribution also agrees completely with that calculated by one PC.

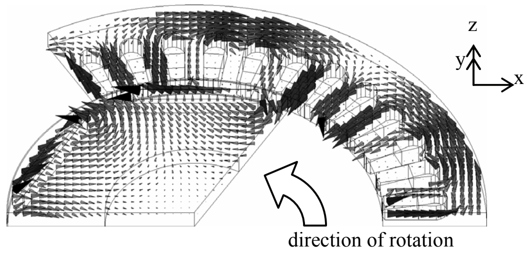


Fig. 4. Distribution of flux density vectors.

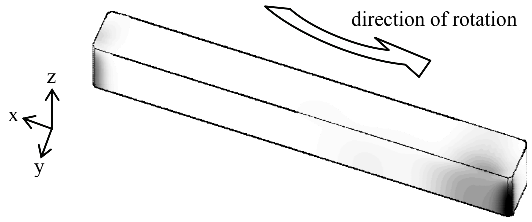


Fig. 5. Distribution of eddy current loss in permanent magnet.

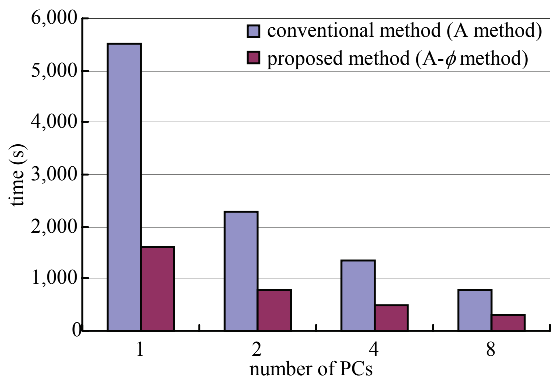


Fig. 6. Calculation time.

The results mentioned above show that the proposed method is appropriate.

Fig. 6 shows the calculation time for one time step. The calculation time with the conventional method employed the A method, is also shown in the same figure. For any given number of PCs, the proposed method is over 2.5 times faster than the conventional method. The speedup ratio of 8 PCs to one PC is 7.2 in the conventional method, and the speedup ratio is 5.6 in the proposed method.

In the full paper, the proposed method will be applied to analysis of a rotating machine excited from voltage sources.

IV. REFERENCES

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